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[5822]-201

F.Y. B.Sc.

MATHEMATICS

MT - 121 : ANALYTICAL GEOMETRY

(2019 Pattern) (Semester - II) (Paper - I) (12111)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following : **[5]**

a) If the origin is shifted to the point (–4, –7), the axes remaining parallel to the original set of axes, find the co-ordinates of the point (–2, 5) with reference to the new set of axes.

b) If the axes are rotated through an angle 60° then write down equations of rotation.

c) If direction ratios of the line are 1, 2, 2 then find its direction cosines.

d) Write equation of a plane parallel to XY-plane.

e) Find equations of the line passing through (2, –3, 5) and whose direction ratios are 1, –2, 2.

f) Find the centre of the sphere
$x^2 + y^2 + z^2 + 2x + 2y - 2z + 3 = 0$.

Q2) a) Attempt any one of the following : **[6]**

i) Reduce the equation $5x^2 + 6xy + 5y^2 - 10x - 6y - 3 = 0$ to its standard form.

ii) Show that the equation
$(ax + by + cz + d) + \lambda (a_1 x + b_1 y + c_1 z + d_1) = 0$ represents the system of planes through the line of intersection of the planes $ax + by + cz + d = 0$ and $a_1x + b_1y + c_1z + d_1 = 0$.

b) Attempt any one of the following : **[4]**

i) Discuss nature of the conic
$5x^2 + 4xy + 3y^2 + 2x + y = 0$. Also find its centre if it is a central conic.

ii) Show that every equation of first degree in x, y, z represents a plane.

P.T.O.

Q3) a) Attempt any one of the following : **[6]**

i) Let the axes be rotated through an angle θ without shifting origin. Let p(x,y) and p(x', y') be the points with respect to original axes and new axes respectively. Show that

$x = x' \cos\theta - y' \sin\theta$, $y = x' \sin\theta + y' \cos\theta$.

ii) Show that equation of a plane which makes intercepts a, b, c on the co-ordinate axes is given by $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

b) Attempt any one of the following : **[4]**

i) Find the co-ordinates of the point of intersection of the line $\frac{x+1}{1}=\frac{y+3}{3}=\frac{z-2}{-2}$ with the plane $3x + 4y + 5z = 5$.

ii) Find equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3).

Q4) a) Attempt any one of the following : **[6]**

i) Find length of the perpendicular from the point (1, 2, 3) to the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$.

ii) Prove that the plane section of a sphere is a circle.

b) Attempt any one of the following : **[4]**

i) Find direction ratios of the line

$ax + by + cz + d = 0$

$a_1x + b_1y + c_1z + d_1 = 0$.

ii) Find the value of 'a' for which the plane $x + y + z = a\sqrt{3}$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$.



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P4715

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F.Y. B.Sc.

MATHEMATICS

MT-122 : Calculus - II

(2019 Pattern) (Semester - II) (Paper - II) (12112)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Five of the following : **[5]**

a) Whether every continuous function is differentiable? Justify

b) State Lagrange's Mean Value Theorem.

c) Determine whether the differential equation $(2x + y)dx + (y + 2x)dy = 0$ is exact or not.

d) Find the general solution of equation $\frac{dy}{dx} = -\frac{x}{y}$.

e) Find an integrating factor, so that $y\,dx - x\,dy = 0$ is an exact differential equation.

f) Evaluate, $\lim_{x \to 1} \frac{\ln x}{x-1}$.

g) Find n^{th} derivative of the function $y = a^{3x}$, $a > 0$.

Q2) a) Attempt any ONE of the following : **[5]**

i) State and prove Cauchy's Mean Value Theorem.

ii) Show that, n^{th} derivative of $y = e^{ax} \cos(bx + c)$ is $y_n = r^n e^{ax} \cos(bx + c + n\theta)$,

where $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$.

P.T.O.

b) Attempt any ONE of the following : **[5]**

i) The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , if\ x \neq 0 \\ 0 & , if\ x = 0 \end{cases}$

show that, f is differentiable at $x = 0$.

ii) By using Taylor's series expansion, expand $x^3 + 7x^2 - 6$ in powers of $(x - 3)$.

Q3) a) Attempt any ONE of the following : **[5]**

i) State and prove Leibnitz theorem for n^{th} derivative of the product of two differentiable functions.

ii) If $p(x)$ is continuous on (a, b) then the general solution of the homogeneous equation $\frac{dy}{dx} + p(x)\,y = 0$ on (a, b) is $y = c.\ e^{-Q(x)}$, where $Q(x) = \int p(x)\,dx$, $a < x < b$.

b) Attempt any ONE of the following : **[5]**

i) Evaluate, $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$.

ii) Solve the differential equation,
$(6xy^2 + 2y)\,dx + (12x^2y + 6x + 3)\,dy = 0$.

Q4) a) Attempt any ONE of the following : **[5]**

i) Explain the method of solving Homogeneous nonlinear equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$.

ii) Define exact differential equation. Explain the method of solving exact differential equation.

b) Attempt any ONE of the following : **[5]**

i) Solve the Bernoulli's equation, $x^2\frac{dy}{dx} + 2xy = y^3$.

ii) Verify Rolle's theorem for the function,

$f(x) = 2x^3 + x^2 - 4x - 2$ on $\left[-\sqrt{2}, \sqrt{2}\right]$.

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